

Parvatibai Chowgule College of Arts and Science
Autonomous

BSc Semester End Examination, January 2022

Semester: I

Subject: Physics

Title: Introduction to Mathematical Physics (Core)

Duration: 2 hours

Max. Marks: 45

Instructions: i) All questions are compulsory.

ii) Use of non-programmable calculators are allowed.

iii) Draw neat diagrams wherever necessary.

Q.1. Answer **Any Three** of the following

[3 + 3 + 3 = 9 Marks]

- a. Explain with an example what do you understand by a Unitary matrix.
- b. Use the ratio test to decide whether the following series is convergent or divergent

$$\sum_{n=0}^{\infty} \frac{5^n (n!)^2}{(2n)!}$$

- c. Find the general solution of the following differential equation

$$y'' + y' - 2y = 0$$

- d. Use differential to find approximately the value of

$$\frac{1}{\sqrt{0.5 - 10^{-20}}} - \frac{1}{\sqrt{0.5}}$$

Q.2. Answer **ANY TWO** of the following.

[6 + 6 = 12 Marks]

- a. Find the first few terms of the Maclaurin series for the following function

$$e^{\sin x}$$

(Hint-Keep terms only till x^4 .)

b. Find the scalar and vector products of the vectors \bar{A} and \bar{B} , where $\bar{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\bar{B} = 4\hat{i} + 2\hat{j} - 3\hat{k}$. Also find the angle between \bar{A} and \bar{B} .

c. Find the general solution of the following differential equation

$$5y'' + 12y' + 20y = 120 \sin 2x$$

Q.3. Answer **ANY TWO** of the following.

[6 + 6 = 12 Marks]

a. Find the general solution of the following differential equation

$$(D^2 + 2D + 17)y = 60 e^{-4x} \sin 5x$$

b. Solve each of the set of equation by finding inverse of coefficient matrix

$$x - y + z = 4$$

$$2x + y - z = -1$$

$$3x + 2y + 2z = 5$$

c. By using the method of expanding function in power series, show that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Also, find the interval of convergence of the above series.

Q.4. Answer **All** of the following

[4 + 4 + 4 = 12 Marks]

a. Find the solution of the following differential equation for x in terms of y

$$dx + (x - e^y)dy = 0$$

b. Express the sin and cosine in exponential form and then evaluate the following integral

$$\int_0^{2\pi} \sin^2 4x \, dx$$

c. Find the eigenvalues of the following 2×2 Hermitian matrix

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

OR

Q.4. Answer **All** of the following

[4 + 4 + 4 = 12 Marks]

p. Express the following in rectangular form $x + iy$
 $\sin(\pi - i \ln 3)$

q. Express $\frac{\partial w}{\partial v}$ in terms of u and v , if $w = xy + yz + xz$ and $x = u + v, y = u - v, z = uv$

r. Using a figure, express the formulas for x, y, z in terms of spherical coordinates r, θ, ϕ and express the spherical unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ in terms of $(\hat{i}, \hat{j}, \hat{k})$. Also, write the expression for the infinitesimal volume element $d\tau$ and find the expression of the volume of a sphere.
